2023
MATHEMATICS

Full marks: 80 Time: 3 hours

General instructions:

i) *Approximately 15 minutes is allotted to read the question paper and revise the answers.*
ii) *The question paper consists of 18 questions. All questions are compulsory.*
iii) *Marks are indicated against each question.*
iv) *Internal and general choice has been provided in some questions.*
v) *Use of simple calculators (non-scientific and non-programmable) only is permitted.*

N.B: Check that all pages of the question paper is complete as indicated on the top left side.

Section – A

1. **Choose the correct answer from the given alternatives:**

(a) Let $f : \text{R} \rightarrow \text{R}$ be defined as $f(x) = 3x$, then **1**

(i) f is one-one onto (ii) f is many-one onto
(iii) f is one-one but not onto (iv) f is neither one-one nor onto

(b) The principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ is **1**

(i) $\frac{-\pi}{3}$ (ii) $\frac{\pi}{3}$ (iii) $\frac{-2\pi}{3}$ (iv) $\frac{2\pi}{3}$

(c) If A, B are symmetric matrices of same order, then AB – BA is a **1**

(i) Skew symmetric matrix (ii) Symmetric matrix
(iii) Zero matrix (iv) Identity matrix

(d) Let A be a square matrix of order 3 × 3. Then $|kA|$ is equal to **1**

(i) $k|A|$ (ii) $k^2|A|$ (iii) $k^3|A|$ (iv) $3k|A|$

(e) If $\sqrt{x} + \sqrt{y} = \sqrt{a}$, then $\frac{dy}{dx}$ is equal to **1**

(i) $-\sqrt{\frac{y}{x}}$ (ii) $-\sqrt{\frac{x}{y}}$ (iii) $\sqrt{\frac{y}{x}}$ (iv) $\sqrt{\frac{x}{y}}$

(f) $\int e^x \sec x(1+\tan x)dx$ is equal to **1**

(i) $e^x \cos x + C$ (ii) $e^x \sec x + C$
(iii) $e^x \sin x + C$ (iv) $e^x \tan x + C$

(g) The order and degree of the differential equation $\left(\frac{ds}{dt}\right)^4 + 3s\left(\frac{d^2s}{dt^2}\right) = 0$ respectively are: **1**

(i) 4 and 2 (ii) 2 and 1 (iii) 1 and 4 (iv) 1 and 2

(h) The probability of obtaining an even prime number on each die, when a pair of dice is rolled is **1**

(i) $\frac{1}{26}$ (ii) $\frac{1}{36}$ (iii) $\frac{1}{46}$ (iv) $\frac{1}{56}$

Section – B

2. Prove: $3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. **2**

3. Find a matrix X such that 2A + B + X = 0, where $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$. **2**

4. Find the equation of line joining (1, 2) and (3, 6) using determinants. **2**

5. Find $\frac{dy}{dx}$, if $x = a(\theta - \sin\theta)$, $y = a(1+\cos\theta)$. **2**

6. Find the intervals in which the function $f(x) = 2x^2 - 3x$ is
(i) increasing (ii) decreasing. **2**

7. Evaluate $\int \frac{\cos x}{\sqrt{1+\sin x}} dx$. **2**

8. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$. **2**

9. The probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that the problem is solved. **2**

Section – C

10. An organization conducted a bike race under two different categories- boys and girls. Totally there were 250 participants, out of which three from category 1 and two from category 2 were selected for the final race. John forms two sets B and G with these participants for his college project. Let $B = \{b_1, b_2\, b_3\}$ and $G = \{g_1, g_2\}$ where B and G represents the set of boys and girls respectively, who were selected for the final race.

Answer the following using the above information. **4**

a). John wishes to form all the relations possible from B to G. How many such relations are possible?

(i) 2^6 (ii) 2^5
(iii) 0 (iv) 2^3

b). John wants to know among those relations, how many functions can be formed from B to G?

(i) 2^2 (ii) 2^3
(iii) 2^{12} (iv) 3^2

c). Let $R : B \to B$ be defined by R = {(x, y): x & y are participants of same sex}. Then R is

(i) Equivalence (ii) Reflexive and symmetric but not transitive
(iii) Reflexive only (iv) Reflexive and transitive but not symmetric

d). Let $R : B \to G$ be defined by R = {(b_1, g_1), (b_2, g_2), (b_3, g_1)}, then R is

(i) one to one (ii) onto
(iii) one to one & onto (iv) neither one to one nor onto

11. **Answer any three from the following questions (a) to (e).** **3 × 4 = 12**

(a) Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x.

(b) Evaluate $\int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx$.

(c) By using the properties of definite integrals, evaluate $\int_0^{\pi/4} \log(1+\tan x) dx$.

(d) Solve the differential equation $(x^2 - y^2)dx + 2xydy = 0$.

(e) Find the particular solution of the differential equation $\frac{dy}{dx} + 2y\tan x = \sin x$ given that $y = 0$ when $x = \frac{\pi}{3}$.

12. **Answer any two from the following questions (a) to (c).** **$2 \times 4 = 8$**

(a) Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$, respectively form the vertices of a right angled triangle.

(b) Find the area of a triangle having the points A(1, 1, 1), B(1, 2, 3) and C(2, 3, 1) as its vertices.

(c) Find the equation of the line in vector and cartesian form that passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction $\hat{i} + 2\hat{j} - \hat{k}$.

13. **a.** Solve graphically: Maximise Z = $3x + 2y$ subject to $x + 2y \le 10, 3x + y \le 15;\ x, y \ge 0$.

Or **4**

b. Solve graphically: Minimise Z = $x + 2y$ subject to $2x + y \ge 3, x + 2y \ge 6;\ x, y \ge 0$.

14. **a.** A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

Or **4**

b. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

Section – D

15. **a.** Solve the system of linear equations by using matrix method.

$$2x + 3y + 3z = 5$$
$$x - 2y + z = -4$$
$$3x - y - 2z = 3$$

Or **6**

b. For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$.

Show that $A^3 - 6A^2 + 5A + 11I = 0$ and hence find A^{-1}.

16. **a.** Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

Or **6**

b. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

17. **a.** Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

Or **6**

b. Find the area enclosed by the circle $x^2 + y^2 = a^2$.

18. **a.** Find the shortest distance between the lines whose vector equations are

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and

$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

Or **6**

b. Find the shortest distance between the lines whose vector equations are

$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$ and

$\vec{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$

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