SCHOOL OF ENGINEERING AND TECHNOLOGY
D.C. COURT JUNCTION, DIMAPUR
END-TERM EXAMINATIONS, DEC 2022

| Course Code: | GIT01 | Semester: | I | Total: | 60 Marks |
| Course Name: | Engineering Mathematics - I | | | Time: | 3 hrs |

(Write only the question number on the answer script)

PART – A

Answer the following questions $\qquad$ $10 \times 1 = 10$

1. The determinant of an Identity matrix of order 10 is:

    (a) 10   (b) 1
    (c) 1/10   (d) 9

2. The eigen values of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ is:

    (a) 1,2,4   (b) 2,2,2
    (c) 2,4,8   (d) no eigen values

3. Which of these convergence tests are for alternating series?

    (a) Limit Comparison test   (b) d'Alembert's Ratio test
    (c) Leibnitz test   (d) Cauchy's Integral test

4. If the series $\sum u_n$ is convergent then:

    (a) $\lim_{n \to \infty} u_n = \infty$ (b) $\lim_{n \to \infty} u_n = 1$
    (c) $\lim_{n \to \infty} u_n = finite$ (d) $\lim_{n \to \infty} u_n = 0$

5. The radius and center of curvature of the curve $(x-2)^2 + y^2 = 25$ is:

    (a) 25, (0,0)   (b) 5, (2,0)
    (c) 25, (−2,0)   (d) 5, (−2,0)

6. Which of the following does not have a defined radius of curvature?

    (a) $y = x^2$ (b) $y = 2x$
    (c) $y = x^2 + 2$ (d) $y = x^3$

7. $\frac{\partial^2}{\partial x \partial y} (x^2 y + y^2 x)$ equals:

    (a) $x + y$ (b) $x^2 + y^2$
    (c) $2x + 2y$ (d) $2xy$

8. Which of the following functions does not have a third derivative?

(a) $y = x^5 + 5$

(b) $y = x^4 + 4$

(c) $y = x^3 + 3$

(d) $y = x^2 + 2$

9. $\int_1^3 \int_2^4 dx\, dy$ is equal to:

(a) 4 sq units

(b) 3 sq units

(c) 2 sq units

(d) 1 sq units

10. $\int_1^3 \int_0^3 \int_2^5 dx\, dy\, dz$ is the same as:

(a) $\int_1^3 \int_2^5 \int_0^3 dy\, dx\, dz$

(b) $\int_1^3 \int_0^3 \int_2^5 dy\, dz\, dx$

(c) $\int_1^3 \int_0^3 \int_2^5 dz\, dy\, dx$

(d) $\int_1^3 \int_0^3 \int_2^5 dx\, dz\, dy$

PART – B

Answer any five questions.

$5 \times 4 = 20$

11. Find the rank of the given matrix A by reducing it to Echelon form.

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ 2 & 5 & 3 & 2 \\ 3 & 2 & 4 & 1 \\ 2 & 4 & 8 & 0 \end{bmatrix}$$

12. Using Cayley-Hamilton theorem find the inverse of A.

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 3 & -3 \\ -2 & -5 & 2 \end{bmatrix}$$

13. Test the convergence of the following alternating series:

(i) $\sum (-1)^{n-1} \frac{1}{n\sqrt{n}}$

(ii) $\sum (-1)^{n-1} \frac{1}{n^3}$

14. Find the center of curvature of the curve $y = x + \frac{9}{x}$ at $(3, 6)$.

15. Discuss the convergence of the given series using Limit Comparison Test

$$\sum \frac{1}{n^2(n+1)}$$

16. Using double integrals find the area of the region bounded by the x axis, y axis and the line $x + y = 1$

17. Evaluate the given integral:

$$\int_{1}^{3}\int_{2}^{3}\int_{0}^{y} (x^2 + y^2 + z^2)dxdydz$$

PART – C

Answer any four questions $4 \times 7.5 = 30$

18. Find the eigen values and vectors of the given matrix

$$A = \begin{bmatrix} 5 & 7 & -5 \\ 0 & 4 & -1 \\ 2 & 8 & -3 \end{bmatrix}$$

19. Test the convergence of the given series using d'Alembert's Ratio Test.

$$\sum \frac{x^n}{n^2 + 1}$$

20. If p_1 and p_2 be the radii of curvature of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the two extremities $(a,0)$ and $(0,b)$ prove that $\left[(p_1)^{\frac{2}{3}} + (p_2)^{\frac{2}{3}} \right](ab)^{\frac{2}{3}} = a^2 + b^2$

21. If $u = \tan^{-1}\left(\frac{y^2}{x}\right)$ show that:

$$x^2\frac{d^2u}{dx^2} + 2xy\frac{d^2u}{dxdy} + y^2\frac{d^2u}{dy^2} = \frac{sin2u}{2}(cos2u - 1)$$

22. Using Leibnitz theorem show that if $y = \sin(m\sin^{-1}x)$ then

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0$$

23. Evaluate $\iint(x^2y + xy^2)dxdy$ over the region bounded by the curves $y = x$ and $y = x^2$.